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Brief communication

Acceleration of a liquid drop suddenly exposed to a high-speed airstream

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Abstract

In this communication we propose a correlation for the drag coefficient on a liquid drop suddenly exposed to a high-speed airstream. The correlation is a bi-power law in the Ohnesorge and Weber numbers. The correlation predicts the acceleration of the drop in terms of known quantities.

Keywords: Drop breakup; High-speed airstream; Drag coefficient; Drop acceleration

1. Introduction

One of the most challenging components in the study of liquid drops suddenly exposed to high-speed airstreams is the measurement of the drop acceleration. This is a consequence of the short time that is available for recording a sufficient number of images of the motion and the related difficulty in obtaining an accurate measurement of the drop displacement as a function of time in the beginning stage of the drop's movement. In the present context we mean by 'high-speed airstreams' flows in which the free-stream velocity is supersonic or high subsonic, which in turn implies large Weber numbers. Thus the discussions in this note do not embrace studies at low subsonic, low Weber number conditions, as for example the work of Hsiang and Faeth (1995), and others.

Studies on the aerodynamic breakup of liquid drops suddenly exposed to high-speed flows date back to the paper by Engel (1958), and several papers appeared in the 1960s and 1970s. The most

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Table 1
Reported drag coefficients and k values

Reference	Liquids	We range	C_D range from acceleration	$K(=\frac{3}{8}C_D)$	Comments
(a) Univ. of Minnesota shock tube					
Joseph et al. (1999)	Newtonian and viscoelastic	10^4–1.7×10^5	1.7–4.0	0.64–1.50	$Ms = 2$ and 3
Brenden (1999)	Viscoelastic	2×10^4–10^5	1.2–3.3	0.45–1.24	$Ms = 2.8$–4.7
Eichman (2001)	Viscoelastic	2×10^4–9×10^4	2.1–3.8	0.78–1.43	$Ms = 2.8$ and 3.5
Ortiz (2003)	Newtonian and viscoelastic	10^3–3×10^4	1.0–3.2	0.38–1.20	$Ms = 3.5$
(b) Other shock tube data					
Reinecke and McKay (1969)	Water	10^4–3×10^5	2.13	0.8	$k = 0.8$ for all experimental conditions $Ms = 3$–12
Reinecke and Waldman (1975)	Water	1700	2.21	0.83	Data at $Ms = 2.3$
Ranger and Nicholls (1969)	Water		2.9	1.1	Average for several experiments. No data on We and Oh
Nicholson (1968)	Water	840–9200	2.67	1.0	Quoted by Pilch and Erdman (1987)
Simpkins and Bales (1972)	Water	34,000	2.14	0.80	Can only compute at 1 point; suggested average $C_D = 2.5$
Engel (1958)	Water	4700	2.21[a]	0.83	Only one C_D value given

[a] Based on intermed. accel.

well-known of these are listed in Table 1. All these studies were carried out in shock tubes, using some form of high-speed recording system to measure initial drop displacement versus time histories. All the investigations were carried out using water drops. Possibly the most accurate acceleration data reported so far can be found in the papers of Joseph et al. (1999), Joseph et al. (2002), and the masters theses of Brenden (1999), Eichman (2001), and Ortiz (2003). These authors studied the breakup of a variety of Newtonian and viscoelastic drops, covering a broad range of viscosities, in the high-speed airstream behind the shock wave in a shock tube. The motion was recorded using a high-speed rotating drum framing camera that can operate up to 200,000 frames/s, thereby giving a time interval of 5 μs between frames. The photographs were scanned into a workstation and composed into movie sequence using commercial software. Displacement versus time graphs were developed from a frame-by-frame analysis of these sequences. Finally, experimental accelerations, $\dot{V}$, were obtained by curve-fitting using a second order polynomial $x - x_0 = \alpha(t - t_0)^2$. All the data for drag coefficients accumulated by the above authors for Newtonian and viscoelastic liquids were used to establish an empirical correlation that relates the drag coefficient to the Weber and Ohnesorge numbers. This correlation may be used to obtain a reasonable prediction for the acceleration, a requirement for predictions of Rayleigh–Taylor instabilities.

2. Dimensionless parameters

The difference between the motion of drops and bubbles in fluids is that a liquid drop falling in air will be subject to Rayleigh–Taylor instability because the heavy fluid accelerates into the light fluid (or vacuum), whereas an air bubble rising in a liquid will not have this instability. In the study of liquid drops suddenly exposed to high-speed airstreams, the drop acceleration is very large, 10^4 or 10^5 times the acceleration of gravity. Acceleration is the principal feature producing Rayleigh–Taylor instabilities which are always in evidence on these drops. The dimensionless parameter for acceleration is the drag coefficient:

$$C_{\mathrm{D}} = \frac{m\dot{V}}{\rho_{\mathrm{a}} A V^2/2} \tag{1}$$

where m is the mass of the drop, V is the air speed relative to the drop, $\dot{V}$ is the drop acceleration, ρ_{a} is the air density, and A is the projected area. The first response of the drop after it is exposed to a high-speed airstream is a flattening of the drop caused by pressure recovery. The drop also accelerates but does not move noticeably; hence V is the airspeed relative to a stationary drop. The displacement versus time curve obtained for the next stage in the breakup process is well fit by a parabola (see, for example, Joseph et al. (1999)), indicating that constant acceleration is a good approximation for the early motion in the breakup process.

It is well known that in steady flow any dimensionless parameter governing drop dynamics may be expressed in terms of two other dimensionless parameters. In drop breakup studies, the Weber number, We, and the Ohnesorge number, Oh, are frequently used:

$$We = \frac{d\rho_{\mathrm{a}} V^2/2}{\gamma}, \quad Oh = \frac{\mu_{\mathrm{l}}}{\sqrt{\rho_{\mathrm{l}}\gamma d}} \tag{2}$$

where d is the initial drop diameter, γ is the surface tension, and μ_{l} is the viscosity of the liquid drop. We propose a functional relationship for the drag coefficient in the form of a bi-power law:

$$C_{\mathrm{D}} = A + K Oh^p We^q \tag{3}$$

The mass of the drop m can be obtained if the volume ϑ of the drop is known. The initial drop diameter is the diameter of a sphere with volume ϑ. Thus, we may write the drag coefficient in terms of the parameter χ:

$$C_{\mathrm{D}} = \frac{\rho_{\mathrm{l}}\chi}{\rho_{\mathrm{a}} V^2/2}\dot{V} \tag{4}$$

where ρ_{l} is the density of the liquid drop, and the parameter χ is defined as follows:

$$\chi = \frac{\vartheta}{A}, \quad \text{for sphere } \chi = \frac{2}{3}d. \tag{5}$$

The acceleration in terms of χ is thus

$$\dot{V} = \frac{\rho_{\mathrm{a}} V^2/2}{\rho_{\mathrm{l}}\chi} C_{\mathrm{D}} \tag{6}$$

3. Previous studies

All of the previous shock-tube studies on the break-up of droplets in high-speed airstreams of the class discussed in this note were carried out using water droplets (Table 1). Thus the roles played by viscosity and viscoelasticity on both the initial acceleration and the drag coefficient could not be identified. In particular, dependence on the Ohnesorge number did not arise, because all experiments were conducted at essentially the same small value of this parameter (0.002–0.003). In these previous works experimenters attempted to obtain data on displacement versus time for the early part of the motion. These data were expressed in dimensionless form, and a curve

$$X = kT^2 \tag{7}$$

was used to fit the data, where dimensionless variables T and X are defined as

$$X = \frac{x}{d}, \quad T = \frac{tV}{d}\sqrt{\frac{\rho_a}{\rho_l}} \tag{8}$$

and t is real time. The constant k in Eq. (7) is a measure of the (constant) acceleration which, in turn, determines a measured drag coefficient:

$$\dot{V} = \frac{d^2x}{dt^2} = 4\left(\frac{\rho_a V^2}{2}\right)\frac{k}{\rho_l d} = 4\frac{We\, k\gamma}{\rho_l d^2} \tag{9}$$

Eq. (6) for spheres is

$$\dot{V} = \frac{3}{2}\frac{We\gamma}{\rho_l d}C_D \tag{10}$$

so

$$C_D = \frac{8}{3}k \tag{11}$$

The values that have been proposed for k in earlier studies (using water) range from 0.8 to 1.1, yielding drag coefficients in the range 2.13–2.93. Pilch and Erdman (1987) collected the then-existing data for drag coefficients in compressible flows (mainly subsonic) and suggested a mean valued 2.5 for the drag coefficient. The prior studies pertinent to this note are listed in part (b) of Table 1, where we have listed only those experimental results for which the authors have provided sufficient data to allow independent calculations of the Weber number and the drag coefficient.

4. Drag coefficient correlation as a function of Weber and Ohnesorge numbers

All the acceleration data from the works listed in part (a) of Table 1 were used to determine the constants A, K, p and q in the functional relationship for C_D (Eq. (3)), yielding the correlation:

$$C_D = 1.6 + 0.4 Oh^{0.08} We^{0.01} \tag{12}$$

Seventy-seven data points were used to establish this correlation. These include data for Newtonian liquids covering a range of viscosities from 0.001 to 10 kg/m s (Ohnesorge numbers from 0.002 to 44) and a variety of polymeric solutions with shear viscosities from 0.04 to 35 kg/m s (Ohnesorge numbers from 0.27 to 82). The shock Mach numbers ranged from 2.0 to 4.7, and free-stream Weber numbers were in the range 1000–162,000.

The predicted acceleration is obtained from Eq. (10) using the predicted C_D from Eq. (12). Predicted accelerations are plotted against measuring accelerations in Fig. 1 for all the data used in deriving Eq. (12).

The average error between predicted and measured acceleration is 23%. It should be remarked that the restriction of 5 μs between frames and limit on measuring displacements to plus or minus one pixel lead to potential errors in the measured accelerations of between 5% and 10%.

Predicted accelerations are compared with experimental accelerations from prior studies in Fig. 2 for cases in which data is available to compute a predicted C_D. The predicted accelerations

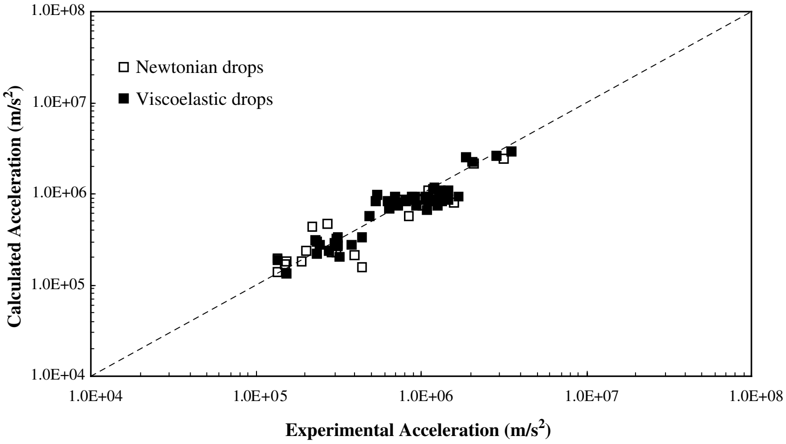

Fig. 1. Calculated acceleration versus experimental acceleration. The correlation works well for Newtonian and viscoelastic liquids.

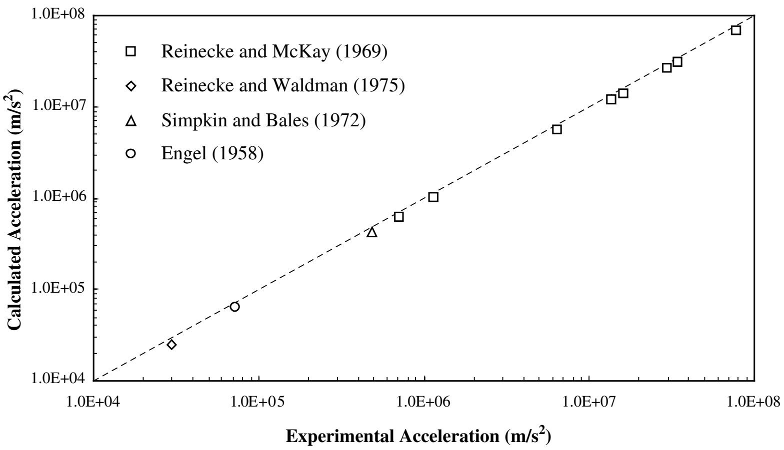

Fig. 2. Calculated acceleration versus experimental acceleration from prior studies.

are consistently about 12% below the measured accelerations. All the data in Fig. 2 were obtained with water ($0.002 < Oh < 0.003$), so Eq. (12) is

$$C_D = 1.6 + 0.25We^{0.01} \tag{13}$$

For this situation CD is essentially independent of Weber number (e.g. $We = 2000$, $C_D = 1.87$; $We = 200,000$, $CD = 1.88$), which leads to a constant value for k of 0.70. This is about 12% less than the value 0.8 used by Reinecke and co-workers (1969, 1975). All the data shown in Fig. 2 were obtained for a supersonic free-stream Mach number with the exception of the data point of Engel.

5. Drag coefficient and Mach number

The correlation (12) has no direct dependence on the free-stream Mach number. However, since the Mach number is an index for the flow upon which the free-stream velocity V and the free-stream density ρ_a depend, and both of these appear in the definition of the Weber number, its indirect effect is already included in the correlation. The data used to obtain the correlation (12) cover a shock Mach number range of $2 < Ms < 4.7$, giving a free-stream Mach number in the range from about 0.95 to 1.63.

The direct effect of the free-stream Mach number on the drag coefficients for different shaped rigid bodies can be seen in Fig. 3, which is reproduced from Howarth (1953). It is important to note the sharp change in drag coefficient when the free-stream Mach number increases from about 0.7 to about 1.5, which is just the range of free-stream Mach numbers that occur in most shock-

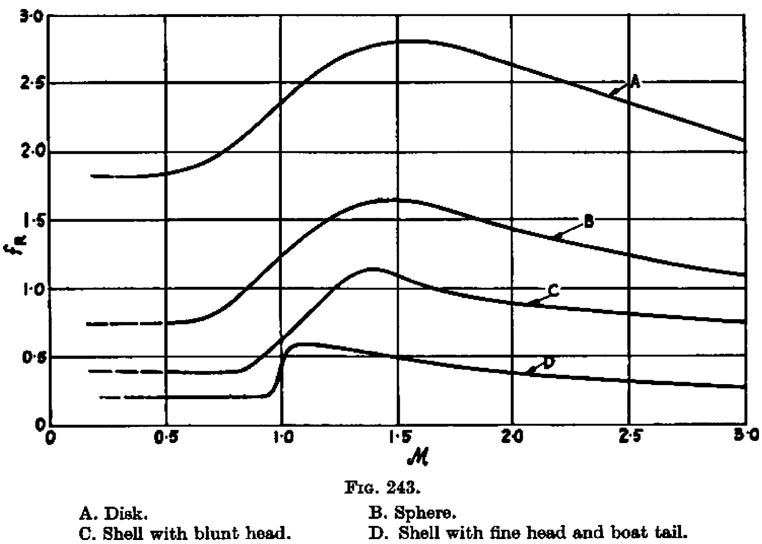

Fig. 3. (Howarth, 1953, p. 724) Effect of Mach number on drag coefficient for four different objects. In this figure f_R stands for $\frac{\pi}{2}C_D$ for spheres and disks.

tube studies. It should be noted that the function f_R in Howarth's figure is $C_D(\pi/2)$ for spheres and disks. Thus for free stream Mach numbers in the range 0.7–1.5 the drag coefficient for rigid spheres is in the range 0.64–1.08, whereas measured values for distorting spheres are of order twice this.

6. Conclusions

The breakup of drops in a high-speed compressible flow with Mach number between about 0.7 to 1.7 is controlled by the free-stream dynamic pressure. The Mach number is an index for these conditions and does not enter explicitly into the correlation for the drag coefficient (12). This correlation approximates the drag coefficient for a large range of Weber and Ohnesorge numbers and applies for both Newtonian and viscoelastic liquid drops. Using this drag coefficient the initial drop acceleration can be predicted to within a certain accuracy. It is of interest that the bi-power law correlation in terms of the Weber and Ohnesorge numbers appears to work as well for viscoelastic drop as for Newtonian drops.

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